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DJ and TK

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1 Model Summary

When market share of an alternative $j = 1, \ldots, J$ in market $t = 1, \ldots, T$ is specified as

$$s_{jt} = \int \frac{\exp(x_{jt}\beta)}{\sum_{k=1}^{J} \exp(x_{kt}\beta)} dF(\beta). \tag{1}$$

The goal is to estimate $F(\beta)$ by setting up discrete points $\beta = \{\beta^{(1)}, \beta^{(2)}, \ldots, \beta^{(R)}\}$ where

$$\beta^{(r)} = \frac{r}{R} \text{ for } 1 \leq r \leq R \tag{2}$$

and each point $\beta^{(r)}$ has its corresponding mass $\bar{f}(\beta^{(r)})$.

The estimation is done by

$$\{\bar{f}(\beta^{(r)})\}_{r=1}^{R} = \text{ argmin}_{\{\bar{f}(\beta^{(r)})\}_{r=1}^{R}} \sum_{t=1}^{T} \sum_{j=1}^{J-1} (\hat{s}_{jt} - \bar{s}_{jt})^2 \tag{3}$$

where

$$\bar{s}_{jt} = \sum_{r=1}^{R} \frac{\exp(x_{jt}\beta^{(r)})}{\sum_{k=1}^{J} \exp(x_{kt}\beta^{(r)})} \bar{f}(\beta^{(r)}) \tag{4}$$

subject to

$$\bar{f}(\beta^{(r)}) \geq 0 \text{ for all } r \tag{5}$$

$$\sum_{r=1}^{R} \bar{f}(\beta^{(r)}) = 1 \tag{6}$$

2 Estimation Excercise

2.1 Data Description

- The data has market share information (MS) for 10 products (J) in 50 markets (T).
- There are two product characteristics $x1$ and $x2$.

2.2 Parameters to Recover

- There are only three types of consumers with different proportions (discrete heterogeneity). Each group i will have $\frac{w_i}{10}, i = 1, 2, 3$ as its proportion where w_i are integers and $w_1 + w_2 + w_3 = 10$
- *The corresponding parameters β_{i1}, β_{i2} for two product characteristics lie on unit interval* $[0.1]$ *and the possible values are* $\{0.1, 0.2, \ldots, 1\}$.
- In sum, you need to estimate one sets of parameters for each of the three groups

$$(\beta_{i1}, \beta_{i2}, w_i), \quad i = 1, 2, 3$$

  resulting in 9 parameters to recover.

2.3 Estimation Tips

- Since each parameter takes a discrete value in $[0.1, 0.2, \ldots, 1]$, you can set up 10 discrete points $[0.1, 0.2, \ldots, 1]$ for each coefficient. Then by combination, you will get 100 basis points $\beta^{(r)}$ on two-dimensional parameter space.
- Optimizing using 'lsqlin' will get the answer perfectly.
- To get the 9 parameters, you need to estimate $\bar{f}(\beta^{(r)})$ for each of 100 basis points. The suggested starting value is 0.01 for these values $\bar{f}(\beta^{(r)})$. However, it does not seem to matter much with 'lsqlin'.